The Memory Effect and What it Means for Humankind

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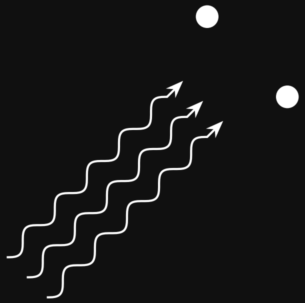
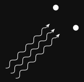
UFABC

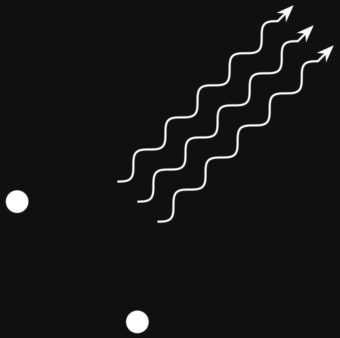
Don’t Forget About Memory



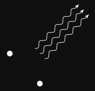
Zel'dovich and A. G. Polnarev 1974. *Sov. Astron.* **18**, p. 17



Consider two inertial detectors “near infinity”. If a gravitational wave passes through them, what happens?



Zel'dovich and A. G. Polnarev 1974. *Sov. Astron.* **18**, p. 17



In general, there will be a permanent displacement between the detectors. This is known as the (linear) memory effect.

How much deviation?

$$\frac{\mathrm{d}^2 \Delta x^j}{\mathrm{d}t^2} = -R_{0i0}{}^{j} d^i$$

$$\frac{d^2\Delta x^j}{dt^2} = -R_{0i0}{}^{j} d^i$$

We can calculate the deviation using the geodesic deviation equation. d^i is the initial deviation, Δx^j is the deviation as a function of time.

$$R_{0i0j} = -\frac{1}{r}\frac{\mathrm{d}^4 Q_{ij}^{\mathrm{TT}}}{\mathrm{d}t^4}(t-r)$$

$$Q_{ij}(t) = \int T^{00}(t, \mathbf{x})x_i x_j \, \mathrm{d}^3 x$$

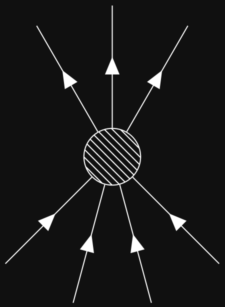
$$R_{0i0j} = -\frac{1}{r}\frac{\mathrm{d}^4 Q_{ij}^{\mathrm{TT}}}{\mathrm{d}t^4}(t-r)$$

$$Q_{ij}(t) = \int T^{00}(t,\mathbf{x})x_i x_j\,\mathrm{d}^3x$$

At large r, the relevant Riemann tensor component can be computed from the transverse-traceless projection of the quadrupole moment. Notice we are ignoring non-radiative modes.

$$\frac{\mathrm{d}^2 \Delta x^j}{\mathrm{d}t^2} = \frac{1}{r} \frac{\mathrm{d}^4 Q^{ij\,\mathrm{TT}}}{\mathrm{d}t^4} d_i$$

The geodesic deviation equation in terms of the quadrupole moment.



Braginsky and Thorne 1987. *Nature* **327**, pp. 123–125

For concreteness, consider the gravitational scattering of a number of bodies. We assume the bodies are asymptotically inertial *in the background Minkowski spacetime*.

$$\frac{\mathrm{d}^2 Q^{ij}}{\mathrm{d}t^2} = \sum_k m_k v_k^i v_k^j$$

Braginsky and Thorne 1987. *Nature* **327**, pp. 123–125

$$\frac{d^2Q^{ij}}{dt^2} = \sum_k m_k v_k^i v_k^j$$

For this scattering setup, the second derivative of the quadrupole moment is given by a simple expression involving the masses and velocities of each body. At early and late times, the third derivative of the quadrupole moment vanishes.

$$\Delta x^j = \frac{d_i}{r} \frac{\mathrm{d}^2 Q^{ij\,\mathrm{TT}}}{\mathrm{d}t^2}\bigg|_{-\infty}^{\infty}$$

Braginsky and Thorne 1987. *Nature* **327**, pp. 123–125

$$\Delta x^j = \frac{d_i}{r} \frac{\mathrm{d}^2 Q^{ij_{\mathrm{TT}}}}{\mathrm{d}t^2}\bigg|_{-\infty}^{\infty}$$

The generic memory deviation.

$$\Delta x^j = \frac{d_i}{r}\left[\sum_{k_f} m_{k_f} v^i_{k_f} v^j_{k_f} - \sum_{k_0} m_{k_0} v^i_{k_0} v^j_{k_0}\right]^{\mathrm{TT}}$$

Braginsky and Thorne 1987. *Nature* **327**, pp. 123–125

$$\Delta x^j = \frac{d_i}{r}\left[\sum_{k_f} m_{k_f} v^i_{k_f} v^j_{k_f} - \sum_{k_0} m_{k_0} v^i_{k_0} v^j_{k_0}\right]^{\mathrm{TT}}$$

The Braginsky–Thorne formula for the memory due to a scattering process.

Note: there is also a nonlinear memory effect

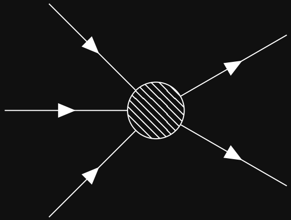
Blanchet and Damour 1992. *Phys. Rev. D* **46**, pp. 4304–4319
Christodoulou 1991. *Phys. Rev. Lett.* **67**, pp. 1486–1489

Note: the nonlinear memory effect can be understood as the memory effect due to the propagation of gravitational waves themselves

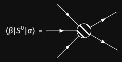
Thorne 1992. *Phys. Rev. D* **45**, pp. 520–524

And Now a Brief Message from Our Sponsor

Infrared Structure of Gravity

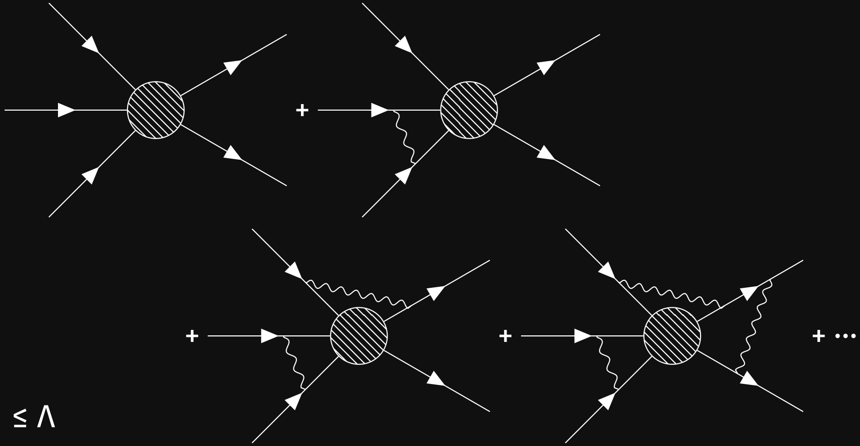
$\langle\beta|S^0|\alpha\rangle =$ 

Weinberg 1965. *Phys. Rev.* **140**, B516–B524



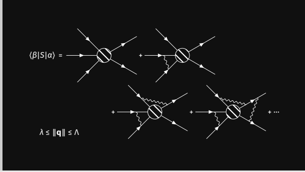
Consider an arbitrary $\alpha \to \beta$ scattering in a theory that includes gravitons. Consider the amplitude for this scattering assuming no soft gravitons are exchanged or emitted.

$$\langle\beta|S|\alpha\rangle =$$



$$\lambda \le \|\mathbf{q}\| \le \Lambda$$

Weinberg 1965. *Phys. Rev.* **140**, B516–B524



Consider now the $\alpha \to \beta$ scattering with the exchange of an arbitrary number of soft gravitons (defined to have momentum $\lambda \leq \|\mathbf{q}\| \leq \Lambda$). Λ defines what is a soft graviton, while λ is an infrared cutoff to show the effect of the divergences.

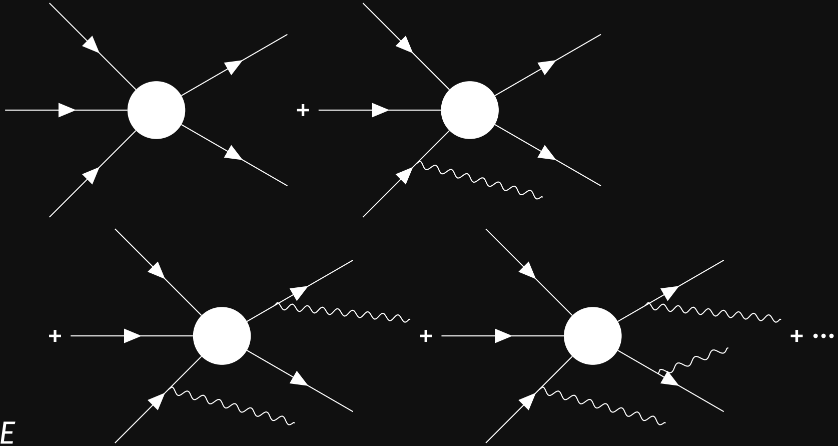
$$|\langle\beta|S|\alpha\rangle|^2 = \left|\langle\beta|S^0|\alpha\rangle\right|^2\left(\frac{\lambda}{\Lambda}\right)^B \xrightarrow{\lambda\to 0} 0$$

Weinberg 1965. *Phys. Rev.* **140**, B516–B524

$$|\langle\beta|S|\alpha\rangle|^2 = \left|\langle\beta|S^0|\alpha\rangle\right|^2\left(\frac{\lambda}{\Lambda}\right)^B \xrightarrow{\lambda\to 0} 0$$

The rate with soft graviton exchanges is simply related to the original transition rate. However, when removing the infrared cutoff, the theory is plagued with infrared divergences that force the rate to vanish.

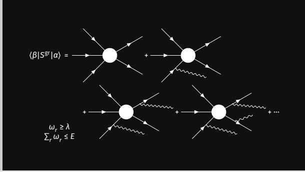
$\langle \beta | S^{\mathrm{gr}} | \alpha \rangle =$



$\omega_r \geq \lambda$

$\sum_r \omega_r \leq E$

Weinberg 1965. *Phys. Rev.* **140**, B516–B524



Consider now the $\alpha \to \beta$ scattering with the emission of an arbitrary number of soft gravitons (defined to have energies $\omega \geq \lambda$ and total summed energy $\sum_r \omega_r \leq E$). E restricts the total energy "lost" to soft gravitons, while λ is an infrared cutoff to show the effect of the divergences.

$$\left|\langle\beta|S^{\mathrm{gr}}|\alpha\rangle\right|^2 = |\langle\beta|S|\alpha\rangle|^2 b(B)\left(\frac{E}{\lambda}\right)^B \xrightarrow{\lambda\to 0} \infty$$

Weinberg 1965. *Phys. Rev.* **140**, B516–B524

$$\left|\langle\beta|S^{\mathrm{gr}}|\alpha\rangle\right|^2 = |\langle\beta|S|\alpha\rangle|^2 b(B)\left(\frac{E}{\lambda}\right)^B \xrightarrow{\lambda\to 0} \infty$$

The rate with soft graviton emissions is simply related to the original transition rate. However, when removing the infrared cutoff, the theory is plagued with infrared divergences that lead the rate to a divergence.

$$\left|\langle\beta|S^{\mathrm{gr}}|\alpha\rangle\right|^2 = \left|\langle\beta|S^0|\alpha\rangle\right|^2 b(B)\left(\frac{E}{\Lambda}\right)^B$$

Weinberg 1965. *Phys. Rev.* **140**, B516–B524

Combining the virtual and real calculations, the infrared divergences cancel out and the inclusive result is finite.

How are these calculations performed?

The dominance of the $1/(p \cdot q)$ pole in (2.5) implies that the effects of attaching one soft-graviton line to an arbitrary diagram is to supply a factor equal to the sum of (2.5) over all external lines in the diagram

$$(8\pi G)^{1/2} \sum_n \eta_n p_n^\mu p_n^\nu / [p_n \cdot q - i\eta_n \epsilon]. \tag{2.7}$$

Weinberg 1965. *Phys. Rev.* **140**, B516–B524

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$$(8\pi G)^{1/2} \sum_n \eta_n p_n^\mu p_n^\nu / [p_n \cdot q - i\eta_n \epsilon]. \tag{2.7}$$

Excerpt from Weinberg 1965 containing the basics of Weinberg's soft graviton theorem, which establishes how the *S*-matrix element for an amplitude with an extra soft graviton relates to the original *S*-matrix element.

In either case, so long as the source is not at a cosmologically large distance, the permanent change in the gravitational-wave field (the burst's memory) δh_{ij}^{TT} is equal to the 'transverse, traceless (TT) part' of time-independent, Coulomb-type, $1/r$ field of the final system minus that of the initial system. If $\mathbf{P}^A$ is the 4-momentum of mass A of the system and P_i^A is a spatial component of that 4-momentum in the rest frame of the distant observer, and if $\mathbf{k}$ is the past-directed null 4-vector from observer to source, then δh_{ij}^{TT} has the following form:

$$\delta h_{ij}^{TT} = \delta\left(\sum_A \frac{4P_i^A P_j^A}{\mathbf{k}\cdot\mathbf{P}^A}\right)^{TT} \tag{1}$$

Here we use units with $G = c = 1$.

Braginsky and Thorne 1987. *Nature* **327**, pp. 123–125

In either case, so long as the source is not at a cosmologically large distance, the permanent change in the gravitational-wave field (the burst's memory) δh_{ij}^{TT} is equal to the 'transverse, traceless (TT) part' of time-independent, Coulomb-type, $1/r$ field of the final system minus that of the initial system. If $\mathbf{P}^A$ is the 4-momentum of mass A of the system and P_i^A is a spatial component of that 4-momentum in the rest frame of the distant observer, and if $\mathbf{k}$ is the past-directed null 4-vector from observer to source, then δh_{ij}^{TT} has the following form:

$$\delta h_{ij}^{TT} = \delta\left(\sum_A \frac{4P_i^A P_j^A}{\mathbf{k}\cdot\mathbf{P}^A}\right)^{TT} \qquad (1)$$

Here we use units with $G = c = 1$.

Excerpt from Braginsky and Thorne 1987 with the original form of the Braginsky–Thorne formula.

The Braginsky–Thorne formula is
a Fourier transform of Weinberg's soft factor!

Strominger and Zhiboedov 2016. *J. High Energ. Phys.* **2016**, 86 . arXiv: 1411.5745 [hep-th]

The Braginsky–Thorne formula is
a Fourier transform of Weinberg's soft factor!

It was pointed out by Strominger and Zhiboedov (2016) that the Braginsky–Thorne formula is related to Weinberg's soft graviton factor by a Fourier transform.

Is there a symmetry behind Weinberg’s soft graviton theorem?

Relations between S-matrix elements are often due to symmetries. Is there a symmetry behind Weinberg's soft graviton theorem?

Bondi–Metzner–Sachs: at infinity in asymptotically flat spacetimes, the group of symmetries is the Poincaré group plus infinitely many supertranslations

Bondi, Van der Burg, and Metzner 1962. *Proc. R. Soc. Lond. A* **269**, pp. 21–52
Sachs 1962. *Proc. R. Soc. Lond. A* **270**, pp. 103–126

Bondi–Metzner–Sachs: at infinity in asymptotically flat spacetimes, the group of symmetries is the Poincaré group plus infinitely many supertranslations

Bondi, Metzner, and Sachs found out in the 1960s that the group of symmetries at null infinity in asymptotically flat spacetimes is not only the Poincaré group, but also includes infinitely many supertranslations. The full infinite-dimensional group is known as the Bondi–Metzner–Sachs (BMS) group.

Weinberg's soft graviton theorem follows from BMS invariance!

Strominger 2014. *J. High Energ. Phys.* **2014**, 152 . arXiv: 1312.2229 [hep-th]
He et al. 2015. *J. High Energ. Phys.* **2015**, 151 . arXiv: 1401.7026 [hep-th]

Weinberg's soft graviton theorem follows from BMS invariance!

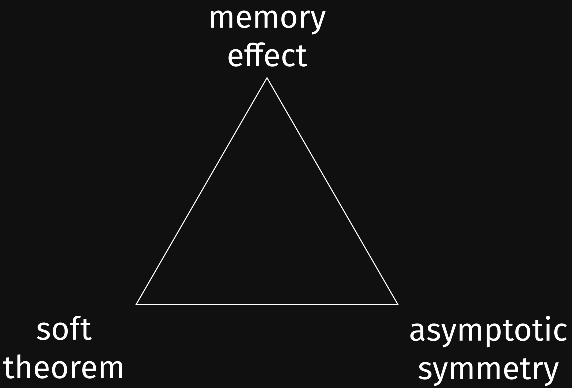
Strominger (2014) and He et al. (2015) showed that Weinberg's soft graviton theorem is actually the Ward identity of the supertranslations.

The memory effect is
a physical realization of a supertranslation!

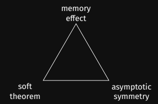
Strominger and Zhiboedov 2016. *J. High Energ. Phys.* **2016**, 86 . arXiv: 1411.5745 [hep-th]

The memory effect is
a physical realization of a supertranslation!

It was shown by Strominger and Zhiboedov (2016) that the displacement due to the memory effect can be understood as a supertranslation acting on the detectors.



Strominger 2018. *Lectures on the Infrared Structure of Gravity and Gauge Theory.* arXiv: `1703.05448 [hep-th]`



There is a relation between the memory effect, the soft graviton theorem, and the BMS symmetries. This triangle echoes through other theories and effects.

Memory and supertranslations may shed light on black hole evaporation

Strominger and Zhiboedov 2016. *J. High Energ. Phys.* **2016**, 86 . arXiv: 1411.5745 [hep-th]
Hawking, Perry, and Strominger 2016. *Phys. Rev. Lett.* **116**, 231301 . arXiv: 1601.00921 [hep-th]
Hawking, Perry, and Strominger 2017. *J. High Energ. Phys.* **2017**, 161 . arXiv: 1611.09175 [hep-th]

Memory and supertranslations may shed light on black hole evaporation

Strominger and Zhiboedov (2016), Hawking, Perry, and Strominger (2016), and Hawking, Perry, and Strominger (2017) argued that supertranslations and the memory effect can shed some light on the loss of information through black hole evaporation.

A Few Comments on Measuring Memory

In binary mergers, the linear memory vanishes because the system is bound. However, the nonlinear effect contributes.

Favata 2010. *Class. Quantum Grav.* **27**, 084036 . arXiv: 1003.3486 [gr-qc]
Grant and Nichols 2023. *Phys. Rev. D* **107**, 064056 . arXiv: 2210.16266 [gr-qc]

The memory is too faint for LIGO to detect it in individual events because it takes time for the memory to build up and the detectors are not inertial. Nevertheless, there are estimates that stacking many events together can lead to a detection in O5

Favata 2010. *Class. Quantum Grav.* **27**, 084036 . arXiv: 1003.3486 [gr-qc]
Grant and Nichols 2023. *Phys. Rev. D* **107**, 064056 . arXiv: 2210.16266 [gr-qc]

LISA has a better chance at measuring memory in individual events because it responds better to low frequencies and it is inertial

Favata 2010. *Class. Quantum Grav.* **27**, 084036 . arXiv: 1003.3486 [gr-qc]
Grant and Nichols 2023. *Phys. Rev. D* **107**, 064056 . arXiv: 2210.16266 [gr-qc]

Further Reading

- Linear and nonlinear memory effects: Bieri and A. Polnarev 2024. *Gravitational Wave Displacement and Velocity Memory Effects.* arXiv: 2402.02594 [gr-qc]
- Relation with soft theorems and asymptotic symmetries: Strominger 2018. *Lectures on the Infrared Structure of Gravity and Gauge Theory.* arXiv: 1703.05448 [hep-th]
- Infrared divergences and soft theorems: Weinberg 1995. *Foundations*

Bieri, Lydia and Alexander Polnarev (2024). *Gravitational Wave Displacement and Velocity Memory Effects*. arXiv: 2402.02594 [gr-qc].

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— (2017). "Superrotation Charge and Supertranslation Hair on Black Holes". In: *Journal of High Energy Physics* **2017**.5, 161. DOI: 10.1007/JHEP05(2017)161. arXiv: 1611.09175 [hep-th].

He, Temple et al. (2015). "BMS Supertranslations and Weinberg's Soft Graviton Theorem". In: *Journal of High Energy Physics* **2015**.5, 151. DOI: 10.1007/JHEP05(2015)151. arXiv: 1401.7026 [hep-th].

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