
silly big questions
can we go back in time?
what are things made of?
what is time?
how did the universe begin?
why do things fall down?

Something I find interesting about theoretical physics is how easily we can get to deep questions about the Universe. Even a toddler can ask questions that we have spent millennia struggling to answer in a satisfying manner. Out of many such questions, today I'm particularly interested in "why do things fall?" Or, more specifically, "why do things fall down?"

Aristotle

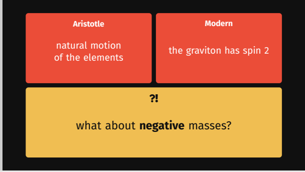
natural motion
of the elements

Modern

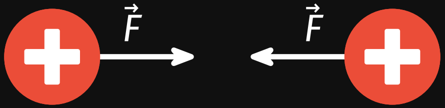
the graviton has spin 2

?!

what about **negative** masses?



According to Aristotle, things would fall down because they are made out of five elements (earth, water, air, fire, and aether) and each of them follows their natural motion. The natural motion of earth and water, for instance, is for them to be closer to the center of the Earth than air or fire. In the modern point of view, we know from general relativity that the graviton has spin 2, and therefore it leads to universally attractive forces. But objects with negative mass could still lead to gravitational repulsion, so why don't we see them?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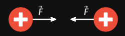
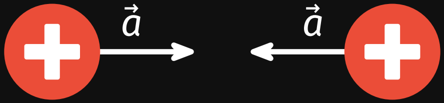
$\vec{F}$



1. To see this effect, first consider the gravitational interaction of two positive-mass bodies.
2. Newton's universal law of gravitation leads to forces pointing toward each other.


$\vec{a}$
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Using Newton's second law we find that positive masses lead to attractive interaction.


$\vec{F}$
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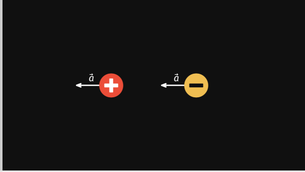
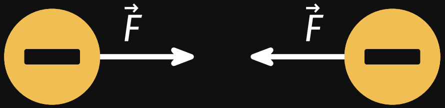
1. Consider next the gravitational interaction of one positive-mass body and one negative-mass body.
2. Newton's universal law of gravitation now leads to forces pointing away from each other.


$\vec{a}$
$\vec{a}$



Mass signs lead to a “pursuit” interaction, because Newton’s second law depends on the mass sign.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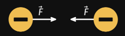


1. Consider finally the gravitational interaction of two negative-mass bodies.
2. Gravity leads to forces pointing toward each other.

$\vec{a}$
$\vec{a}$

Negative masses lead to repulsive interaction.

?!

but where are the negative masses?

energy conditions

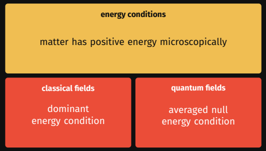
matter has positive energy microscopically

classical fields

dominant energy condition

quantum fields

averaged null energy condition



To understand why we don't see negative masses, we can look to energy conditions. These conditions are often imposed in general relativity in order to restrict attention to "physically acceptable" forms of matter. They usually say that energy is microscopically positive in some suitable sense

1. Known forms of classical matter satisfy the dominant energy condition, which states that energy densities are always positive and energy fluxes are always causal
2. Quantum fields are believed to satisfy some form of the averaged null energy condition, which states that the sum of energy density and pressure integrate to a non-negative value along a null geodesic

positive mass theorem

isolated bodies
in classical physics
have positive mass

Schoen and Yau 1979. *Commun. Math. Phys.* **65**, pp. 45–76
Schoen and Yau 1981. *Commun. Math. Phys.* **79**, pp. 231–260
Witten 1981. *Commun. Math. Phys.* **80**, pp. 381–402

positive mass theorem

isolated bodies
in classical physics
have positive mass

It is known that in classical general relativity isolated bodies satisfying the DEC must have positive mass. This essentially means that known forms of classical matter lead to positive mass bodies

Penrose–Sorkin–Woolgar theorem

many isolated bodies emerging from QFT have positive mass

Penrose, Sorkin, and Woolgar 1993. arXiv: gr-qc/9301015

Penrose–Sorkin–Woolgar theorem

many isolated bodies emerging from QFT have positive mass

It is known that bodies satisfying the ANEC have positive mass (assuming auxiliary mathematical conditions). This essentially means that quantum matter is expected to lead to positive mass bodies

Aristotle

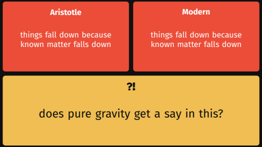
things fall down because known matter falls down

Modern

things fall down because known matter falls down

?!

does pure gravity get a say in this?



Both Aristotelian physics and modern physics state that things fall down because they are made of things that fall downCan we get a result that does not depend so much on what things are made of?

Positive Mass in General Relativity Without Energy Conditions

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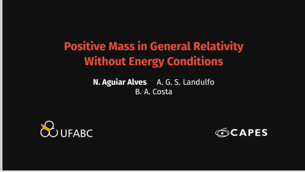
B. A. Costa

UFABC CAPES

Aguiar Alves, Landulfo, and Costa 2024. arXiv: 2408.00154 [gr-qc]



1. In other words, can we get positive mass in general relativity without energy conditions?
2. This was addressed by me in collaboration with André Landulfo and Bruno Arderucio Costa
3. I am currently a second-year PhD student at the Federal University of ABC
4. And during this project I was supported by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior—Brasil (CAPES)—Finance Code 001

what are the simplifying assumptions?

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To make the calculations feasible, we need to make some simplifying assumptions. Which ones did we employ?

1

spherical symmetry

3

near equilibrium

5

barotropic fluid

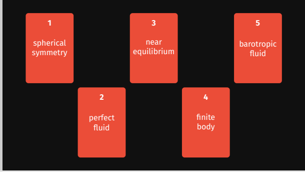
2

perfect fluid

4

finite body

Aguiar Alves, Landulfo, and Costa 2024. arXiv: `2408.00154 [gr-qc]`



1. We assume the solution is spherically symmetric. This implies the stress tensor describes a fluid and that the metric is quite simple
2. We assume the the fluid is a perfect fluid, to avoid dealing with different radial and tangential pressures
3. We assume the solution is “near” a stationary solution, so the system can attain equilibrium
4. We assume matter is compactly supported, so the spacetime describes an isolated finite body
5. We assume the fluid is barotropic as a technical condition

This essentially means we are trying to describe a negative-mass star what is the geometry of a star in equilibrium?

Tolman–Oppenheimer–Volkoff

nonlinear
ODE system

Tolman 1939. *Phys. Rev.* **55**, pp. 364–373
Oppenheimer and Volkoff 1939. *Phys. Rev.* **55**, pp. 374–381

what if the star is not in equilibrium?

Chandrasekhar

linear PDE
for small deviations

Chandrasekhar 1964. *Phys. Rev. Lett.* **12**, pp. 114–116
Chandrasekhar 1964. *ApJ* **140**, pp. 417–433

how do we study the stability
of a negative-mass star?

1

choose a stellar model

2

solve the TOV equation

3

study the Chandrasekhar equation

!

all barotropic models considered are unstable

Aguiar Alves, Landulfo, and Costa 2024. arXiv: 2408.00154 [gr-qc]

To do a case-by-case analysis, we can take the following steps

1. We first choose a specific stellar model (by picking either an equation of state or an energy density profile)
2. We solve for the model's pressure and gravitational field by solving the TOV equations
3. We study the perturbations by analyzing the Chandrasekhar equation
4. All barotropic models considered lead to instabilities

?

what causes the instability?

1

unstable if $(\partial P/\partial \rho)_s < 0$ somewhere

2

this is always the case for barotropic stars!

Aguiar Alves, Landulfo, and Costa 2024. arXiv: 2408.00154 [gr-qc]

What characterizes these instabilities?

1. $(\partial P/\partial \rho)_s < 0$ always leads to instabilities
2. Negative-mass barotropic stars always have $(\partial P/\partial \rho)_s < 0$ somewhere in the interior

all barotropic
negative-mass stars
are unstable!

all barotropic negative-mass stars are unstable!

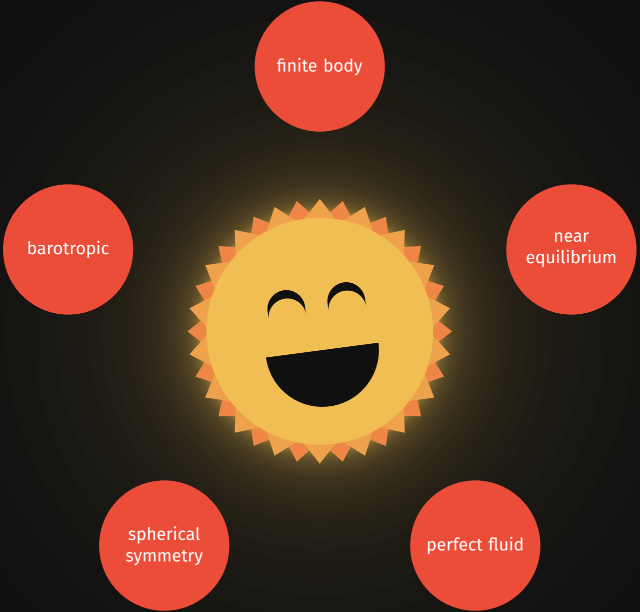
We have thus shown that all barotropic negative-mass stars satisfying our assumptions are hydrodynamically unstable what should you take home?

Negative masses are unstable

and we don't need energy conditions to prove it

Negative masses
are unstable
and we don't need energy conditions to prove it

The main message is that negative masses are unstable and we don't need energy conditions to prove it



Aguiar Alves, Landulfo, and Costa 2024. arXiv: 2408.00154 [gr-qc]

Our result was obtained by considering negative-mass stellar models satisfying some simplifying assumptions. Namely

1. spherical symmetry
2. made of perfect fluid
3. near hydrodynamical equilibrium
4. finite in size
5. made of barotropic fluid

Aguiar Alves, Landulfo, and Costa 2024. arXiv: 2408.00154 [gr-qc]

These conditions imply the negative-mass star is unstable

And hence it will depart from equilibrium

All previous explanations assumed that mass is positive in small scales, but we didn't need to!

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Positive Mass in General Relativity
Without Energy Conditions

arXiv: 2408.00154 [gr-qc]

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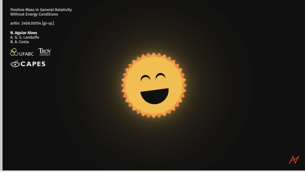

Thank you very much.